A further note on the bathtub vortex

By D. L. KELLY, B. W. MARTIN AND E. S. TAYLOR
Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 11 July 1963)

Using identical equipment, an attempt was made to reproduce the unexpected flow reversals obtained by Sibulkin (1962) in his study of the vortex motion associated with draining a liquid from a vessel through an orifice in its bottom. Reversal of the initial direction of rotation was found only for counter-clockwise filling with large initial heads and settling times, unless during settling or draining of the vessel a shock was applied to the system, in which case the tendency to reversal was generally more pronounced with clockwise filling. Re-reversal of the motion was also observed in a number of cases. An alternative explanation is proposed, based on a consideration of the surface waves generated by a shock.

1. Introduction

In 1962 Sibulkin reported the results of an investigation of the vortex motion which occurs when water is being drained from a bathtub through a hole in its bottom. By observing the motion of a thin cork disk floating on the surface of water contained in a cylindrical transparent vessel 12 inches in diameter, Sibulkin found that when the disk had centred itself on the axis of the vortex created during the emptying process, its direction of rotation reversed during the final stages of emptying before the water surface broke. Reversal was observed for either initial direction of rotation, but over a limited range of settling times between 5 and 20 min. Sibulkin advanced a tentative explanation for the reversal, based on Helmholtz's theorem, in terms of the acquisition by the fluid in the boundary-layer adjacent to the bottom of the tank of a vertical component of the vorticity vector as the outlet is approached. Associated with this vertical component is an induced circulation of opposite sign to that formerly prevailing, so that the direction of rotation is reversed.

With a view to gaining further insight into the factors which might govern such reversal, apparatus was constructed identical in every way with that described by Sibulkin (see figure 1 of his paper) and, using similar techniques, observations were made which generally covered the ranges of conditions mentioned in his note. The results are described below. Sibulkin's nomenclature is used wherever possible.

2. Observations and discussion

When the tank was filled in such a way that, looking down on the apparatus, the initial direction of rotation was clockwise, no reversal of the vortex motion was observed even after exhaustive repetitions of some fourteen tests. These covered initial water heights (z_0) up to 4 in. and settling times (t_s) up to 1 h. Vortex reversal was, however, observed when the initial direction of rotation was made counter-clockwise, but only for $z_0 \geqslant 2$ in. and $t_s \geqslant 20$ min. Reversal then occurred over the range of water heights z_s between zero and 0·9 in.

Quite fortuitously, and due originally to a footstep near the apparatus, it was found that vortex reversal could be induced in many cases where no reversal had previously been noted by the application of either a random or a controlled shock to the bedplate on which the equipment was mounted, thus imparting a disturbance to the fluid. The controlled shock was brought about by dropping a known weight from a known height at a given distance from the centre of the tank. Its magnitude, measured in terms of foot-pounds, was found to be significant. It was also found that the timing of the shock was important. As a result, tests were conducted in which the shock was applied at various times both before and after discharge of the liquid through the orifice had commenced.

From the results obtained under controlled shock conditions, with initially clockwise rotation, in which the respective effects of settling time t_s, initial head z_0, and shock timing t_1 were investigated, it seems that there are optimum conditions of all three variables in the occurrence of vortex reversal. Sibulkin refers to an optimum settling time in his experiments of between 5 and 20 min.; in the present tests the optimum range was between 20 and 30 min. Likewise, vortex reversal was observed only for a range of initial water heights z_0 between 2 and 3 in. Furthermore, the phenomenon was most marked for shock timings immediately before or immediately after discharge began.

There are other interesting features associated with reversal of the vortex motion. First, there was clear evidence in many tests of re-reversal, so that, before the water surface broke, the original direction of rotation was restored. It is difficult to account for this further phenomenon purely on the basis of Sibulkin's proposed explanation. It might, of course, be argued that his explanation does not, and was not intended to, take account of the disturbance created by a shock, were it not that re-reversal was also observed in the tests already mentioned with initially counter-clockwise rotation, when no shock was applied.

Comparison of the present results with those obtained by Sibulkin indicated that the variations of angular displacement θ and angular velocity ω followed broadly similar trends, but numerical magnitudes measured by the authors were larger by a factor of about 2. This may well be due to differing initial circulations, there being no reason to suppose that in the two sets of experiments the water was injected into the tank during the filling process with the same velocity. Sibulkin's data implies a negative and nearly constant angular acceleration at zero angular velocity; in the present tests it was observed that when $\omega = 0$ the angular acceleration was also zero. In many of the tests the period of zero rotation was as much as 6 sec. The fact that in Sibulkin's experiments, ω appears to have reached a stationary value when the water surface broke, raises speculation as to whether or not re-reversal might not otherwise have subsequently occurred.

For initially counter-clockwise flow, as mentioned above, vortex reversal and re-reversal were observed without the application of a shock for relatively large initial heads and long settling times. The application of a shock in these cases generally inhibited reversal (and therefore re-reversal) completely. As z_0 and t_s were reduced to smaller values, reversal could at first be induced by applying a shock, but it become unobtainable with further reduction in these parameters.

The present investigation suggests that a shock is one method by which the vortex motion in a vessel from which liquid is being drained may be modified. This suggestion is supported by further tests in which shocks of a more random and less violent nature were applied, and compared with tests conducted under shock-free conditions, for initially clockwise rotation. In the great majority of these tests a random shock caused vortex reversal, but not re-reversal. It therefore appears possible that relatively small disturbances, brought about by (perhaps unnoticed) vibrations transmitted to the liquid through a structure from a quite extraneous source, may be partly or even wholly responsible for the observed cases of vortex reversal, including those herein reported for initially counter-clockwise rotation, ostensibly without shock. Further consideration is given to these questions below.

3. Alternative explanation of reversal

Consider motion in a cylinderical tank initially of the free vortex type. If undisturbed, the motion will decay due to shear stresses, particularly those at the centre of the vortex and near the boundaries. If there were no boundary friction the free vortex motion would decay into rigid body rotation. With a bottom boundary layer only, the resulting flow will probably be similar to that investigated by Boedewadt and reported by Schlichting (1955), where the boundary layer, which turns out to have a constant thickness, is a small proportion of the total depth. The effect on the Boedewadt solution of the boundary layer on the side wall of the tank will presumably be small.

What will happen if such a flow is allowed to discharge through a central hole? Note that, in the absence of friction, the smaller the angular momentum of a fluid particle, the easier it will be for such a particle to find its way to the central hole. Thus in this sense the central hole is selective, separating fluid on the basis of angular momentum, and giving priority of discharge to fluid particles with smaller angular momenta. The result of such selection will be to discharge first those particles originally at small radii, together with some fluid from the bottom boundary layer.

As discharge proceeds, fluid from increasingly large radii will approach the orifice, and motion more nearly like a free vortex will be restored, first at the centre of the tank but subsequently spreading to larger radii. The angular velocity of fluid particles outside the boundary-layer will steadily increase, as will be evidenced by an increasing rate of rotation of the cork float. Under these circumstances more and more of the fluid discharged will come from the boundary-layer on the bottom of the tank, which now will possess less angular momentum than the fluid above. Finally, the rate of rotation of the cork float will decrease when the water level is so low that the cork float is exposed to fluid in the boundary layer.

On this hypothesis, we would therefore expect the following sequence of events, confirmed both by Sibulkin and the authors: initially, an increasing rate of rotation of the cork float, followed by a steady reduction during the latter stages of the emptying process.

If we observe a reversal in the direction of rotation of the float, it means that part of the fluid has in some way acquired angular momentum of the opposite sense to that gained during filling of the tank. Vortex reversal requires the application of a corresponding reverse torque. It seems unlikely that this is produced solely by the purely dissipative torque due to friction, which, while capable of reducing the motion to zero, cannot, it is thought, cause vortex reversal. Once a particle has reached zero angular velocity, the torque from the wall ceases. We must therefore seek some mechanism by means of which angular momentum can be imparted to at least a portion of the fluid subsequent to filling.

It was observed during the present tests that surface waves were generated by the application of a shock, and it is suggested that they offer one such possible mechanism. Consider a wave system in the tank which consists of a reciprocating motion at the lowest natural frequency. This will cause a back-and-forth displacement of fluid particles as required to satisfy continuity. If a second wave system at right angles to the first and out of phase by one-quarter of a cycle is then superimposed, the fluid particles will move in closed curves approximately circular near the centre of the vessel. (Such motion was in fact observed.) Since all fluid particles move in the same sense round their paths, the fluid as a whole must possess a net angular momentum. Any disturbance (or disturbances) capable of creating the above type of circular motion will therefore add angular momentum to the system. The process by which this is finally converted to circulation is not at all clear, but it would seem as if it must involve frictional forces. Furthermore, to accord both with the above explanation and with the observed sequence of reversal and re-reversal, such circulation must be confined to fluid initially remote from the centre of the vessel and outside the bottom boundary-layer.

REFERENCES

SCHLICHTING, H. 1955 *Boundary-Layer Theory*. New York: McGraw-Hill.
SIBULKIN, M. 1962 *J. Fluid Mech.* **14**, 21.